2011

A1. –

2. (a) Eg $(3 \quad -10 \quad 0)^T$ and $(0 \quad 0 \quad 1)^T$, no; (b) -17.

3. (a) $\begin{pmatrix} 3 & 2 & 0 \\ 2 & 2 & 2 \\ 0 & 2 & 1 \end{pmatrix}$; (b) No, as the other eigenvalues are 5 and -1.

4. (b)(ii) $\frac{-2}{1+ln3}$.

5. $A = C = 0, B = 2$.

6. (a) $k = \left(k_0 - \frac{\beta}{\alpha}\right) e^{\frac{\alpha t^2}{2}} + \frac{\beta}{\alpha}$, steady state is $k = \frac{\beta}{\alpha}$.

7. (NB: you need a Taylor Series around $\bar{y}$, not zero!) $E(U) = 2^{\bar{y}} + 3^{\bar{y}}$,
$var(U) = (2^{\bar{y}} ln2 + 3^{\bar{y}} ln3)^2 \sigma^2 / n$.

8. (a) $\binom{S}{y} \theta^y (1-\theta)^{S-y}$; (b) $K\theta^2(1-\theta)^8$ where K=normalising constant, MLE=0.2;
(c) $K\theta^{10}(1-\theta)^{30}$.

9. (a) $cn^{-1}, n^{-1} + n^{-2}$; (b) $c < 1 + n^{-1}$.

B10. (a) one unique solution if $t \neq -8$, no solutions if $t = -8, s \neq -16$, infinitely many solutions if $t = -8, s = -16$; (b) (ii) f^1, f^4; (iii) f^1: $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, f^4: $\begin{pmatrix} 1 & 1 & 0 \\ 2 & 0 & -1 \end{pmatrix}$.

11. (a) (i) $\begin{pmatrix} 1-b & a \\ m & -h \end{pmatrix} \begin{pmatrix} Y \\ r \end{pmatrix} = \begin{pmatrix} I^0 + G \\ M^S - M^0 \end{pmatrix}$, unique solution requires $h(1-b) + am \neq 0$;
(ii) $Y = \frac{h(I^0+G)+a(M^S-M^0)}{h(1-b)+am}$, $r = \frac{m(I^0+G)-(1-b)(M^S-M^0)}{h(1-b)+am}$; (iii) $\Delta Y = \frac{h\Delta G}{h(1-b)+am}$, $\Delta r = \frac{m\Delta G}{h(1-b)+am}$.

12. (a) $L_1 = \frac{L_0}{\left(\frac{p_1 a_1}{p_2 a_2}\right)^{\frac{1}{b-1}}+1}$, $L_2 = \frac{L_0}{\left(\frac{p_2 a_2}{p_1 a_1}\right)^{\frac{1}{b-1}}+1}$; marginal increase in national output when L_0 increases is $\lambda = \frac{p_2 a_2 b L_0^{b-1}}{\left(\left(\frac{p_2 a_2}{p_1 a_1}\right)^{\frac{1}{b-1}}+1\right)^{b-1}}$; (c) $\frac{dY}{dp_1} = a_1 L_1^b$.

13. (b) (i) (Take "with support (0,4]" to mean $0 < w \leq 4$): $K = \frac{1}{4}$; (ii) 1; (iii) $\frac{4}{3}$; (c) $\frac{1}{4}$.

14. (b) (i) $n = \frac{\sigma_X^2}{\delta \varepsilon^2}$; (ii) $n = 10^5$; (c) $Prob\left(\left|N(0, \frac{\sigma_X^2}{n})\right| \leq \varepsilon\right)$; (d) Via Normal approx: $n > 96$, via Chebyshev: $n \geq 500$.

15. (a) $(1-\theta)y + \theta y^2, \frac{1}{2} + \frac{\theta}{6}$; (b) $\hat{\theta} = 6\bar{y} - 3$; (c) $\sum_i \frac{-1+2y_i}{(1-\theta)+2\theta y_i} = 0$.

2012

A1. $\mathbb{R}^n: Row(A), Null(A), Col(A^T); \mathbb{R}^m: Row(A^T), Null(A^T), Col(A);$
$Col(A) = Row(A^T), Col(A^T) = Row(A)$ otherwise they are distinct.

2. (a) eigenvector is $\boldsymbol{e}_n$.

3. -

4. (a) $\frac{P(data|H_0)P(H_0)}{P(data|H_1)P(H_1)}$; (b) $\frac{8}{3}$.

5. (c) $\alpha = e^{-12\lambda}(1 + 12\lambda + 72\lambda^2)$.

6. (a) $\hat{\theta} = \sum \frac{x_i}{n}$, asymptotic distribution $N\left(\theta, \frac{\theta^2}{n}\right)$, large sample 95% CI is $\frac{\bar{x}}{1 \pm \frac{1.96}{\sqrt{n}}}$.

7. (a) ∞.

8. (a) $\pi = QP(Q^*) - c(Q)$, FOC $P(Q^*) + Q^*P'(Q^*) - c'(Q^*) = 0$, Yes unless $c''(Q) < 0$;
(b) (i) decreases (ii) increases.

9. (a) Standard Lagrangian is $\ln x + m - \lambda(xp + m - w) + \mu m$, Kuhn-Tucker Lagrangian is $\ln x + m - \lambda(xp + m - w)$; (b) $w \geq 1: x = \frac{1}{p}, m = w - 1, w \leq 1: x = \frac{w}{p}, m = 0$;
(c) $w \geq 1: 1, w \leq 1: \frac{1}{w}$.

B10. (a) $Y = \frac{\lambda(I_0+G-aT)+(c+b)(M_S-M_0)}{\lambda(1-a)+(c+b)\mu}, r = \frac{\mu(I_0+G-aT)-(1-a)(M_S-M_0)}{\lambda(1-a)+(c+b)\mu}$; (b) $\Delta G \geq a\Delta T$.

11. -

12. -

13. (a) $\tilde{\sigma}^2 = \frac{\sum x_i^2}{n}, \frac{n\tilde{\sigma}^2}{\sigma^2}$ is χ_n^2 and it is the MVUE (CRLB is $\frac{2\sigma^4}{n}$); (b) $\frac{\sigma^2}{2n}$; (c) $\frac{\sigma^2}{n}\left(\frac{\pi}{2} - 1\right)$, asymptotic efficiency $\frac{1}{\pi-2}$.

14. -

15. (a) $b'\left(b^{-1}(B)\right) = \frac{v-B}{b^{-1}(B)}$; (c) $b(v) = \frac{v}{2}$.

2013

A1. $c = 3, \lambda = 3$ eigenvector $(0 \quad 0 \quad 0 \quad 0 \quad 1)^T, \lambda = 5$ eigenvector $(0 \quad 0 \quad 0 \quad 2 \quad -1)^T$.

2. Inverse is $\begin{pmatrix} I_n & 0 \\ -C & I_m \end{pmatrix}$.

3. (a) independent; (b) dependent; (c) dependent; (d) independent.

4. (a) $\hat{\theta} = -\frac{n}{\sum \ln x_i}$, $N(\theta, \theta^2/n)$, mean = $\frac{\theta}{\theta+1}$, $\hat{\theta} = \frac{\bar{x}}{1-\bar{x}}$.

5. (a) variance-covariance matrix is $\sigma^2(X'X)^{-1}$.

6. (a) $M_z(t) = (pe^t + q)^{2n}$, Z is a binomial $(p, 2n)$.

7. (a) concave; (b) 4!, -5!; (c) $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$.

8. (b) $e - 2$.

9. (a) if $w \leq 20, f = d = \frac{w}{2}$, if $w \geq 20, f = d = 10$; (b) $\frac{w^2}{20} - w$.

B10. (a) $(-0.645 \quad -0.717 \quad -2.222)^T$; (b) 4, no: a change in demand for manufacturing has no impact on demand for tourism.

11. -

12. (a) $\frac{\sigma^2}{n}\chi_n^2$, variance is $2\sigma^4/n$; (b) $S^2 = \frac{1}{n-1}\sum(x_i - \bar{x})^2$; (c) var $(\tilde{\sigma}^2) = \frac{2(n-1)\sigma^4}{n^2}$; $\text{MSE}(S^2) = \frac{2\sigma^4}{n-1}$, $\text{MSE}(\tilde{\sigma}^2) = \frac{(2n-1)\sigma^4}{n^2}$, difference is $\text{MSE}(\tilde{\sigma}^2) = \frac{(3n-1)\sigma^4}{n^2(n-1)}$; (d) $\begin{pmatrix} \frac{n}{\sigma^2} & 0 \\ 0 & \frac{n}{2\sigma^4} \end{pmatrix}$, $\bar{X}$ does attain CRLB but S^2 does not in finite samples.

13. (a) $\binom{145}{x}\theta^x(1-\theta)^{145-x}$; (c) (i) (0.2278, 0.2025, 0.2827,0.2869); (ii) 0.7721.

14. (b) Yes; (c) No; (d) No.

15. (a) $r_{n+1} = (1 - 2\epsilon)r_n - \epsilon p_n + \epsilon$, $p_{n+1} = (1 - \epsilon)p_n + \epsilon r_n$; (b) $v^* = \frac{1}{3}\begin{pmatrix}1\\1\end{pmatrix}$, eigenvalues = $1 - \frac{3\epsilon}{2} \pm \sqrt{3}\epsilon i/2$.

2014

A1. (a) True; (b) False; (c) False.

2. (a) No; (b) x, x^2.

3. -.

4. (b) Poisson, with parameter $\sum \lambda_i$.

5. (a) $\frac{P(M_1|D)}{P(M_0|D)} = \frac{P(D|M_1)}{P(D|M_0)} \frac{P(M_1)}{P(M_0)}$.

6. (a) mean = π, variance = $\frac{\pi(1-\pi)}{n}$.

7. -.

8. (c) Yes.

9. Question should say $\bar{w} = \frac{3}{4}$ (see Examiner's Report) (b) $w = h = \frac{3}{4}$; (c) $-\frac{3}{32}$.

B10. (a)(i) $(1 \quad 0 \quad 1 \quad 2)^T$, $(0 \quad 1 \quad 0 \quad 0)^T$, $(0 \quad 0 \quad 0 \quad -1)^T$; (ii) $(1 \quad 0 \quad 1 \quad 2)^T$, $(0 \quad 1 \quad 0 \quad 0)^T$, $(2 \quad 0 \quad 2 \quad 3)^T$; (iii) $(-1 \quad 0 \quad 1 \quad 0)^T$; (c) Well, zero seems to be the smallest eigenvalue, so $(-1 \quad 0 \quad 1 \quad 0)^T$?

11. (a) $\begin{pmatrix} 1-\tau-s & 0 & f \\ \tau & 1-\sigma & \phi \\ s & \sigma & 1-f-\phi \end{pmatrix}$; (b) $\begin{pmatrix} \frac{100}{366} & \frac{211}{366} & \frac{55}{366} \end{pmatrix} = (0.273 \quad 0.577 \quad 0.150)$; (c) everybody becomes employed with tenure.

12. (b) $\mathrm{E}(\hat{\sigma}_n^2) = \frac{n-1}{n}\sigma^2$, $\mathrm{E}(S_n^2) = \sigma^2$, $\mathrm{var}(\hat{\sigma}_n^2) = \frac{2(n-1)}{n^2}\sigma^4$, $\mathrm{var}(S_n^2) = \frac{2}{n-1}\sigma^4$, $\mathrm{MSE}(\hat{\sigma}_n^2) = \frac{2n-1}{n^2}\sigma^4$, $\mathrm{MSE}(S_n^2) = \frac{2}{n-1}\sigma^4$, $\hat{\sigma}_n^2$ preferable in terms of MSE; (c) (ii) MSEs: $\hat{\theta}_2 : 2\theta - \theta^2$, $\hat{\theta}_n : \frac{\theta(1-\theta)}{n}$, $\hat{\theta}_{n+1} : \frac{n\theta(1-\theta)+\theta^2}{(n+1)^2}$.

13. (a) $\hat{\theta} = \frac{\bar{y}}{\gamma}$; it is the MVUE, variance = $\frac{\theta^2}{n\gamma}$;
(b) $\ln L = \sum \left(-\gamma\beta x_i + (\gamma - 1)\ln y_i - y_i e_i^{-\beta x_i} - \ln \Gamma(\gamma) \right)$.

14. (a) No; (b) π; (c) No; (d) Unclear: depends on the thickness of the paint.

15. (a)(i) $P(t) = \bar{K} + (P(0) - \bar{K})e^{-t}$; (ii) $P(t) \to \bar{K}$; (b)(i) $P = K = 0$ unstable, $P = K = \bar{K}$ stable.

2015

A1. (a) $a=1$; (b) Yes for $S1$, No for $S2$; (c) 3.
2. (a) No; (b) Yes; (c) I'd say yes, though the official answer was no.
3. (a) No; (b) No.
4. (b) $8x10^{-5}$.
5. See 2009 A6 !
6. Yes.
7. $A=e^{e-1}$, $B=1-e^{e}$; (b) $\left(e^{e+1}-2e+e^{1-e}\right)/2$.
8. -.
9. (a) $\partial r*/\partial q=-1/2$, $\partial r*/\partial\delta=-1/2$; (b) $\partial q*/\partial\gamma>0$, $\partial q*/\partial\delta>0$.
B10. (b) $\left(2 \quad 4 \quad -1\right)^{T}$; (c) Yes, $\dim\tilde{V}+\dim\tilde{V}^{\perp}=\dim\mathbb{R}^{3}$.
11. (a) $\begin{pmatrix} o \\ r \end{pmatrix}_{t+1}=\begin{pmatrix} 0.4 & 0 \\ 0.6 & 0.9 \end{pmatrix}\begin{pmatrix} o \\ r \end{pmatrix}_{t}$; (c) $\rho=0.1$, 1 in 6.
12. See 2009 B13 !
13. (a) Quantile function is $\xi=(1-\tau)^{-1/\alpha}$, median ($\tau=1/2$) = 2; (b) $\tilde{\alpha}=\bar{y}/\bar{y}-1$;
(c) $\hat{\alpha}=n/\sum\ln y_i$, α^2/n ; (d) $\ln n-\ln\sum\ln y_i\pm1.96/\sqrt{n}$;
(e) $-2\ln\lambda=2(3+n)\sum\ln y_i+2n+2n\ln n$, so compare this to χ_1^2 in a one tailed test, yes.
14. (b) $g_1x'+g_2y'+g_3z'$.
15. (a) $y=2e^{x}+e^{-x}$; (b) $z_n=2(1+\delta)^{n}\begin{pmatrix} 1 \\ 1 \end{pmatrix}+(1-\delta)^{n}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$; (c) first limit is α .