MANAGEMENT STUDIES TRIPOS

Wednesday 30 April 2008 9.00 - 12.00

Paper M2

QUANTITATIVE METHODS AND OPERATIONS MANAGEMENT

*Answer **four** questions, **two** from Section **A** and **two** from Section **B**.*

*Answers to Sections **A** and **B** must appear in two separate booklets.*

*All eight questions carry 25 marks each. The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

N.B. THE FINAL TWO PAGES OF THIS EXAMINATION PAPER CONSIST OF SPECIAL DATA SHEETS

STATIONERY REQUIREMENTS
2 x 20 Page Booklets
Rough Work Pads

SPECIAL REQUIREMENTS
Approved calculators allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A

1 A Spanish airline with operations at Heathrow Airport, London, must staff its ticket counters inside the airport. Ticket attendants work six-hour shifts at the counter. There are two types of agents – those who speak English as a first language and those who are fully bilingual (English and Spanish). The airline believes that the need for ticket agents between the hours of 6am and 9pm are as in the table below:

	6am - 9am	9am - noon	noon - 3pm	3pm - 6pm	6pm - 9pm
Agents needed	12	20	16	24	12

Agents begin work either at 6am, 9am, noon, or 3pm. The shifts are designated as shifts A, B, C, and D, respectively. It is the policy of the airline that at least half of the agents working in any time period speak English as a first language. Further, at least one-quarter of the agents needed in any time period should be fully bilingual. English speaking agents and bilingual agents are paid £12 and £16 per hour respectively.

(a) Formulate a linear program (LP) to minimize total agent costs. You are <u>not</u> required to solve your optimization problem. [14 marks]

(b) Based on the LP formulation you obtained in part (a), explain what is meant by sensitivity analysis and describe the information that could be gained from a sensitivity analysis with respect to the hourly cost of bilingual agents. [5 marks]

(c) The airline also wants to have a minimum of 6 bilingual agents in shift A, if the total number of agents hired in shift A is no more than 15. Formulate an integer linear program that takes this additional consideration into account in your model in (a). You are <u>not</u> required to solve your optimization problem. [6 marks]

2 (a) Robert Lewis is planning to raise funds for Soccer Aid Charity by running a concession stand during an international game between England and Italy on 1st May 2008. Robert needs to decide whether to rent a large insulated thermos from a local rental store and to sell cocoa at the game, or to rent a large refrigerated container and to sell lemonade. Unfortunately, Robert does not have the resources to rent both items. Sales will depend on whether it is sunny or rainy during the game. If the weather is sunny, Robert will make a profit of £600 from lemonade but only £300 from cocoa. If, however, it is rainy, Robert will make a profit of £800 from cocoa but only break even if he brings lemonade. Based on the local newspaper's prediction, Robert thinks there is a 60% chance of it being sunny on 1st May 2008.

(i) Draw the decision tree for Robert's problem and show what he should do based on the criterion of the expected monetary value. Support your answers with calculations. [8 marks]

(ii) Should Robert change his decision in (i) if the probability of it being sunny had been 70% or 80%? Explain your answer. [4 marks]

Robert's friend Joanna is a meteorologist who claims she can predict the weather more accurately than the newspaper. For only £4, she offers to study the weather and to tell Robert if there is a "good chance" or "bad chance" of it being sunny on 1st May 2008. Assume the following data about Joanna's information are available:

The probability that she will say "good chance" is 0.7 (and therefore the probability that she will say "bad chance" is 0.3).

If she says "good chance", then there is a 0.83 probability that it will actually be sunny on 1st May 2008.

If she says "bad chance", then there is only a 0.25 probability that it will actually be sunny on 1st May 2008.

(iii) How much is Joanna's information actually worth? Support your answer with a decision tree and calculations. [8 marks]

(b) Briefly describe Monte Carlo simulation, and why it might be useful in this case. [5 marks]

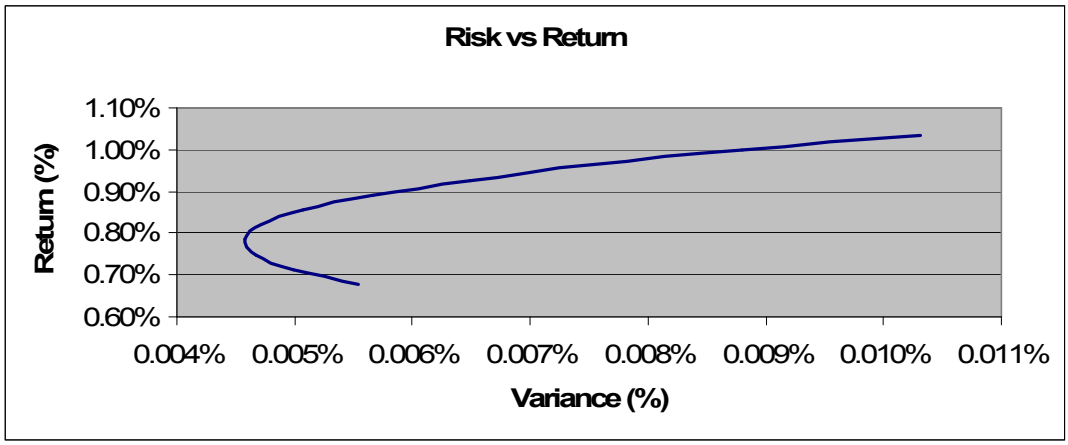
3 (a) You are considering a portfolio made up of shares in two stocks, Hi-Tech (stock symbol X) and Conventional (stock symbol Y). You have 100 months of price data for these stocks.

(i) After converting the stock prices to returns, in the form of monthly percentage changes, the following averages are computed over the 100 months of data:

	X	Y	X squared	Y squared	XY
Average	1.0337%	0.6778%	0.0210%	0.0101%	0.0092%

Calculate the variance for a portfolio which is split 50-50 between Hi-Tech and Conventional. Show brief details of this calculation. (Hint: when finding the product of two percentages, convert the result back into a percentage.) [9 marks]

(ii) The risk-return chart for portfolios that combine Hi-Tech with Conventional in various proportions, for that 100 month period, is:



Use the chart to estimate the range of returns, as percentages to two decimal places, that you would accept for a portfolio that has a risk of not more than 0.005%. Note that your estimates are not expected to be very accurate. [4 marks]

(b) Consider a Markov chain that models the change in the crystalline state of a chemical compound in a reactor vessel, that occurs every 5 minutes. There are four states A, B, C and D and the transition probability matrix for changes of state is given below:

To	From			
	A	B	C	D
A	0%	0%	10%	0%
B	10%	0%	0%	0%
C	0%	40%	90%	10%
D	90%	60%	0%	90%

(i) If you start with 100% of the compound in state A, what percentage of state B will be present after one 5 minute stage? What percentage of state B will there be after two stages? [4 marks]

(ii) Write down the equation that describes the share of A at an equilibrium, assuming an equilibrium exists. Explain briefly what this equation represents. [4 marks]

(iii) Will this Markov chain converge to an equilibrium? Explain briefly your reasoning. [4 marks]

4 (a) You are discussing, with the sales manager of a retail company, how sales levels are related to the marketing budget. Describe briefly, how you would explain to the manager the assumptions that underlie linear regression. [5 marks]

(b) The management of Hotel El-Ay, in Los Angeles, California, wants to gain a better understanding of demand for fully paid advance bookings. It believes that the number of (fully paid advance) bookings depends on the number of quotes given via its booking system, and the "competitive price" of rooms. The competitive price is the average price (in dollars) for equivalent rooms quoted by five local hotels that compete for customers with Hotel El-Ay. The price quoted by Hotel El-Ay for an advance booking depends on both the number of unfilled rooms and the competitive price, on the day of the quote.

You are given 194 days of data for single room bookings. The data available to you includes only the competitive price for single rooms on each day of the data set, not the competitive price that was used to give an advance booking quote on any earlier day. The regression analysis (with the number of fully-paid advance bookings as the dependent variable) gives the following results:

Regression Statistics	
Multiple R	0.846
R Square	0.716
Adjusted R Square	0.713
Standard Error	7.757
Observations	194

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	25.344	4.888	5.185	0.000	15.702	34.986
Number of quotes	0.086	0.007	13.258	0.000	0.073	0.099
Competitive price	0.156	0.095	1.635	0.104	-0.032	0.344

(i) Explain briefly to hotel management the meaning of the above "Multiple R" value. [5 marks]

(ii) How many bookings would you expect for a particular day when there are 1000 price quotes and the competitive price on that day is $36? How would you present this to hotel management? [5 marks]

(iii) What does the coefficient corresponding to "Number of quotes" tell us? What additional information could you give the management about this coefficient? [5 marks]

(iv) How reliable do you consider the regression model summarised in the table above to be? Explain briefly to hotel management *either* why the model should be trusted, *or* any deficiencies that may affect its reliability and what might need to considered to rectify these. [5 marks]

SECTION B

5 Consider the following excerpt from *The Unofficial Guide to Walt Disney World* by Bob Sehlinger.

> Jungle Cruise
>
> *What It Is:* Outdoor safari-themed boat-ride adventure
> *Scope & Scale:* Major attraction
> *Author's Rating:* A long-enduring Disney masterpiece that continually increases in popularity
> *Duration of Ride:* 8–9 minutes
> *Average Wait in Line per 100 People Ahead of You:* 4½ minutes
> *Assumes:* 10 boats operating
> *Description and Comments*: A cruise through jungle waterways. Passengers encounter elephants, lions, and a menacing hippo. Boatman's spiel adds to the fun.

(a) Assuming that the above description refers to when the Jungle Cruise ride is operating at full capacity, calculate this capacity in terms of people per hour. [5 marks]

(b) Assume that, in addition to the ride duration of 8½ minutes, each of the ten boats also spends 1½ minutes of time at the dock, in order to allow one boatload of passengers to disembark and the next group of passengers to board. Calculate the average number of people who will fit into each boat. State any assumptions that you make. [10 marks]

(c) It is currently late September 2008 and Disney World would like to forecast the weekly demand for the Jungle Cruise for the next three months. What forecasting technique would you recommend them to use? Be as specific as you can. [5 marks]

(d) What is the underlying model for the forecasting technique you recommended in part (c)? Be as specific as you can. [5 marks]

6 The book *The Goal,* describes operations at a manufacturing plant.

(a) Briefly describe three specific improvements, other than the use of priority tags, that are recommended to be made at the plant described in *The Goal* and the reasons for each. [5 marks]

(b) Explain the two principal themes, identified in *The Goal*, that need to be focused on in order to optimise the use of bottlenecks? [4 marks]

(c) According to cost accounting rules, capacity is to be balanced with demand, and the aim is to maintain the flow. In *The Goal* this is said to be incorrect. What alternative rule is it argued should be followed here? [4 marks]

(d) What does *The Goal* say about the applicability of the Economic Order Quantity (EOQ)? Discuss, briefly, whether you agree with his claim. [5 marks]

(e) Compare and contrast the general approach to inventory taken in *The Goal* with that of the Toyota Production System. [7 marks]

7 (a) Explain the difference between a product and a service. [4 marks]

(b) Give two examples of products that, depending on the circumstances, might be considered as a service. [4 marks]

(c) Describe the three essential elements of Total Quality Management according to Shewhart. [5 marks]

(d) Describe briefly two areas in which quality management concepts might be used to improve health care and explain how these concepts might be applied in the treatment of patients. [12 marks]

8 (a) Give five reasons for holding inventory. Each reason should be stated in four words or less. [5 marks]

(b) A student drinks an average of five bottles of CPA (Cambridge Pale Ale) each week. A six-pack of CPA sells for £8.00 at an off-licence 2 miles down the motorway from their accommodation. They calculate that, with maintenance, petrol, depreciation, etc., it costs them 24p per mile to drive their car. Their marginal rate of return on investment (i.e., their opportunity cost of capital) is 10.4% per year. Given the data above, calculate how often the student should buy CPA. [10 marks]

(c) The Friends of Friends (FoF) is a political advocacy group. As is common in such organisations, FoF maintains a separate account for its rather high levels of cash disbursements. Cash disbursements occur randomly; that is, they show no particular pattern within, or between, months. Total cash disbursements average about £112,500 per month. The administrative cost of selling any amount of marketable securities – which is where their liquid funds are generally kept — and transferring the proceeds into this cash disbursement account is £20. While the marketable securities tend to earn 12% per year, the account for disbursements only earns 3% annually. How often should FoF transfer money into this special account for cash disbursements? [10 marks]

END OF PAPER

SPECIAL DATA SHEET 1
Formula sheet

Covariance, Correlation and Regression

Consider data pairs (X_1, Y_1), (X_2, Y_2), …, (X_n, Y_n).

Let m_X and m_Y denote the respective means of the X and Y data.

Let s_X and s_Y denote the respective standard deviations of the X and Y data.

Covariance is given by

$$\mathrm{cov}(X,Y) = \frac{\sum_{i=1}^{n}(X_i - m_X)(Y_i - m_Y)}{n} = \frac{\sum_{i=1}^{n} X_i Y_i}{n} - m_X m_Y$$

The correlation coefficient is given by

$$\mathrm{correl}(X,Y) = r = \frac{\mathrm{cov}(X,Y)}{s_X s_Y}.$$

The line of best fit is given by

$$Y - m_Y = \frac{r s_Y}{s_X}(X - m_X).$$

Now consider three random variables x, y and z with means m_x, m_y, and m_z, respectively; variances Var[x], Var[y], and Var[z], respectively; and covariance between x and y, for example, given by

$$\mathrm{cov}(x,y) = \mathrm{E}[(x - m_x)(y - m_y)] = \mathrm{E}[xy] - m_x m_y .$$

Given any numbers α_x, α_y, α_z, let $v = \alpha_x x + \alpha_y y + \alpha_z z$. Then the variance of v is given by

$$\begin{aligned}\mathrm{Var}[v] = {} & \alpha_x^2 \mathrm{Var}[x] + \alpha_y^2 \mathrm{Var}[y] + \alpha_z^2 \mathrm{Var}[z] \\ & + 2\left(\alpha_x \alpha_y \mathrm{cov}(x,y) + \alpha_y \alpha_z \mathrm{cov}(y,z) + \alpha_x \alpha_z \mathrm{cov}(x,z)\right)\end{aligned}$$

Markov Chains

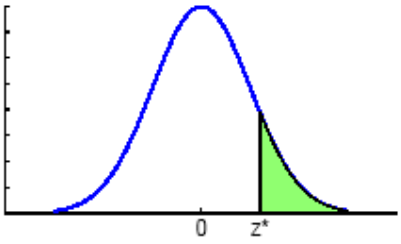
For a Markov chain with 3 states A, B and C, and probabilities of going from one state B to another A written P(B,A), equilibrium shares a, b, c must satisfy

$$\begin{aligned} a &= a P(A,A) + b P(B,A) + c P(C,A) \\ b &= a P(A,B) + b P(B,B) + c P(C,B) \\ c &= a P(A,C) + b P(B,C) + c P(C,C) \\ 1 &= a + b + c \end{aligned}$$

SPECIAL DATA SHEET 2

Standard Normal Distribution Table

(Areas under the standard normal curve beyond z*, i.e., shaded area)



z^*	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010