Answers to MST Paper M2 Quantitative Methods Past Tripos Questions

Below are my answers to the past exam questions for MS2.

If you think I've got any of these wrong then let me know - it's possible I've made an arithmetic slip somewhere, particularly on the more recent years, as those answers have been checked by fewer students.

Some notes: I've done the hypothesis tests using Z values rather than by construction of a confidence interval, as this is more standard statistical practice. When asked to estimate values of the dependent variable from a regression model, I do not always estimate the confidence interval, because you have not been taught how to do this properly - the ±2(standard error) calculation is potentially simplistic. More generally, when constructing a 95% confidence interval, I usually use 1.96 rather than the less precise value of 2.

2003:
1 (a) (i) correlation coefficient = 0.82; $R^2 = 0.68$ (ii) no difference (b) (i) 2.9 (ii) 92%
2 (a) Trump, £52300 (b) get £1.09 back for each £1 invested (c) get £54600 back (d) investing £48932 in Flex and £1068 in the bank gives you £54581 back.
3 (a) (i) mean=11.4, not clear which stdev they want, but σ_n=5.4 σ_{n-1}=5.5 (ii) mode=9.5 to 12.5 bin median=9.5 to 12.5 bin (iii) 24.5%, 97.8% (b) mean=29 stdev=3.7
4 (a) Z=2.15 so 2002 is significantly different at 5% level, two tailed test (b) 1600 (?)

2004:
1 (a) (i) Mean=1.25 Median=1.2 to 1.3 bin (ii) 1.29 (estimated variance is 0.039) (b) Correct method (was not in the MS2 syllabus, at least at that time): pooled estimate of standard deviation is 2.3 and 1.645(2.3) is 3.86, so clearly not significant. Approximate MS2 method: tricky in that a two-tailed test at the 5% significance level would lead to the conclusion that the fall was not significant (CI is 52± 3.3) whereas a one-tailed test at the 5% significance level would lead to the conclusion that the fall was significant (CI is 52±2.7).
2 (a) keep for 3 years NPV=£5198; successive annual replacement NPV=£5968 (b) (i) Option 1 expected cost = £1900; option 2 expected cost=£3450 (ii) Not a good measure - outcomes are too diverse (stdev of expected cost=2801).
4 (a) (i) Y = -25000 + 867X, 25000, correlation coefficient=0.922 (b) (ii) hard to judge the numbers when reading off the graph, but about £36k and £38k

2005:
1 (a) (ii) Seems very complicated, but the best I can do is: introduce an integer variable N_A, with $N_A = 1$ if we do not invest in A, and $N_A = 0$ if we do invest in A. Add conditions $N_A \leq 1$, N_A integer, $(A - 50000) + \alpha N_A \geq 0$, $A <= \alpha(1-N_A)$, where α is a large positive number, as well as the usual non-negativity constraint $N_A \geq 0$; (b) (i) Yes, up to an extra 266.7 hours (ii) Yes, 40 beds and 30 desks, profit=£2250
2 (a) (i) EVs are £42k, £44k, £22k so go for Office (ii) £28k (b) £9442.48
3 (a) (i) weight = -358 + 7.4(height) $R^2 = 0.84$ (ii) with a simplistic ±2 std err logic: 138 ± 18 (iii) 86 (though this is outside the data range?) (iv) I'd say 7.4±3.7, but different simplifying assumptions could produce different answers (b) r=0.94 (ii) charges = 11.7 + 6(people) = 35.7
4 (a) (ii) 0.038 (iii) 0.008 (b) (i) Z=0.71 so insufficient evidence of an improvement (ii) if you think null hypothesis is 45% then n=1057, if you think it is 39.5% then 1021.

2006:

1 (c) introduce integer variable M20, with M20 = 1 if we do run production line I for more than 20 hours, and 0 otherwise. Add conditions M20 $\leq$ 1, M20 integer, (HoursII - HoursI) + αM20 $\geq$ 0 where α is a large positive number, as well as the usual non-negativity constraint M20 $\geq$ 0.
2 work in £million: (a) £1.08 (b) £1.039 (c) III gives £1.032, so is best (d) £0.015, ie £15k.
3 (a) assuming Route-204 is typical of Birmingham, 0.83 (b) assuming 2005 sample was very large, Z=-1.25 so one cannot conclude service has deteriorated at the 5% significance level (c) assuming the time period is one month, 12±1 (d) 0.015 ie 1.5%.
4 (a) r=0.778 (i) with a simplistic ±2 std err logic: 43 ± 33 (ii) Yes (b) Maths mark = 82.5 - 4.0(Absences) R^2 = 0.93 (ii) No.

2007:

1 (c) (i) Z=0.94 so yes (ii) not clear what assumption one should make about the stdev, but if one takes 200 then Z=1.82 so yes; using stdev=295 gives same conclusion (iii) 0.24 ie 24%
2 (c) with a simplistic ±2 std err logic: 55±2(5.79) (d) with a simplistic ±2 std err logic: $55\pm2(5.79)/\sqrt{100}$ (e) -4.466 to -1.700 (f) I'd say 10.9±2.6, but different simplifying assumptions could produce different answers
3 (a) expected costs in \$billion are 4.8, 3.44, 6.04 (c) ignoring the likelihood that the fight in court element of "negotiate patiently" would change as well, the fight in court directly becomes more attractive if prob of losing < 0.34
4 (a) (ii) introduce integer variable e, with e = 1 if we do need the expert, and 0 otherwise. Add conditions e $\leq$ 1, e integer, [Rural0-30] + [Rural31-60] + [Rural61+] - 500 $\leq$ αe where α is a large positive number, as well as the usual non-negativity constraint M20 $\geq$ 0, and then add 1000e to the cost function (b) (i) increase by 5500 (ii) decrease by 700 (iii) no change.

2008:

1 (a) Call the decision variables A_E (number of English speakers on shift A), A_{ES} (number of bilingual English-Spanish speakers on shift A) etc. Then we seek to minimise:

$$12(A_E + B_E + C_E + D_E) + 16(A_{ES} + B_{ES} + C_{ES} + D_{ES})$$

subject to:

$A_E + A_{ES} \geq 12$
$A_E + A_{ES} + B_E + B_{ES} \geq 20$
$B_E + B_{ES} + C_E + C_{ES} \geq 16$
$D_E + D_{ES} \geq 12$
$A_E \geq 0.5(A_E + A_{ES})$
$A_E + B_E \geq 0.5(A_E + A_{ES} + B_E + B_{ES})$
$B_E + C_E \geq 0.5(B_E + B_{ES} + C_E + C_{ES})$
$C_E + D_E \geq 0.5(C_E + C_{ES} + D_E + D_{ES})$
$D_E \geq 0.5(D_E + D_{ES})$
$A_{ES} \geq 0.25(A_E + A_{ES})$
$A_{ES} + B_{ES} \geq 0.25(A_E + A_{ES} + B_E + B_{ES})$
$B_{ES} + C_{ES} \geq 0.25(B_E + B_{ES} + C_E + C_{ES})$
$C_{ES} + D_{ES} \geq 0.25(C_E + C_{ES} + D_E + D_{ES})$
$D_{ES} \geq 0.5(D_E + D_{ES})$

(c) Create a new integer variable $I_{A>15}$, which is equal to zero if $A_E + A_{ES} \leq 15$ and equal to one otherwise. Then:

$A_E + A_{ES} - 15I_{A>15} > 0$ (forces $I_{A>15}$ to be zero if $A_E + A_{ES} \leq 15$, otherwise has no impact)
$I_{A>15} \leq 1$ $I_{A>15} \geq 1$ $I_{A>15}$ integer
$A_{ES} + 6I_{A>15} \geq 6$ (from $A_{ES} - 6(1 - I_{A>15}) \geq 0$

2 (a) (i) go for cocoa, EV = £500; (ii) 70%: no; 80%: go for lemonade; (iii) Joanna's advice is worth £51.70.
3. [Whole question seems problematic to me] (a) (i) I suspect a problem with the figures in the question! (ii) 0.71% to 0.85% (???) (b) (i) 10%, zero
4. (b) (ii) 117

2009-:
Official answers available via CamTools :-)

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